Grid Size Dependence on Convergence for Computation of the Navier-Stokes Equations

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Introduction

A NEW efficient procedure for the numerical solution of Navier-Stokes equations, using line Gauss-Seidel and Newton iterative methods was recently presented by MacCormack.[1] The numerical procedure was applied to the compressible viscous flow of a two-dimensional flow within a transonic converging-diverging nozzle. A steady-state solution was obtained in 16 iterations for a stretched nonuniform 20×20 mesh.

Although the present method showed very high numerical efficiency, the fact that the grid size might severely affect convergence was questionable. It was suggested that the number of iterations would vary directly with the number of grid points. To answer this question, the effect of grid size on convergence was tested by refining the grid size by factors of two and four for the same transonic problem presented by MacCormack.

Governing Equations and Numerical Algorithm

The basic governing equations and numerical algorithm are briefly reviewed. For convenience, they will be described in two-dimensional Cartesian coordinates. The Navier-Stokes equations written in two-dimensional conservation law form are

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0 \tag{1}$$

where elements of U are the conservative variables and F and G the flux vectors. By differentiating Eq. (1) with respect to t and letting $\Delta t(\partial U/\partial t)^n = \Delta U^n$ and $\Delta t(\partial U/\partial t)^{n+1} = \delta U^{n+1}$, an implicit difference approximation is obtained as

$$\left[I + \Delta t\left(\frac{D \cdot A}{\Delta x}\right) + \Delta t\left(\frac{D \cdot B}{\Delta y}\right)\right]\delta U_{i,j}^{n+1} = \Delta U_{i,j}^{n} \tag{2}$$

where $A = \partial F/\partial U$, $B = \partial G/\partial U$, and $D\cdot$ indicates the difference operator. Equation (2) contains a block penta-diagonal matrix. Since there is no efficient direct inversion procedure available, two choices can be taken: either an approximate factorization to convert the left-hand side of Eq. (2) into two block tridiagonal matrices or an indirect iterative inversion. The error of approximate factorization for solving Eq. (2) limits the time step in that the solution can be advanced during each iteration. On the other hand, the indirect line Gauss-Seidel iterative procedure used in this study is not as limited and promises high numerical efficiency.

Test Runs

Three test cases were chosen to answer the question of the effect of grid size on convergence using the line Gauss-Seidel relaxation technique for solving the Navier-Stokes equations. The grid used in Ref. 1 was a stretched nonuniform 16×25 mesh. In the present study mesh was exponentially stretched to provide resolution in the boundary-layer and nozzle throat regions. The grids shown in Fig. 1 with sizes 20×20, 40×40, and 80×80 were used. A converged solution was obtained in 16 iterations on a 20×20 mesh. Test runs were than made on the 40×40 and 80×80 mesh using the same time step as used on the 20×20 mesh. Steady-state solutions were also obtained

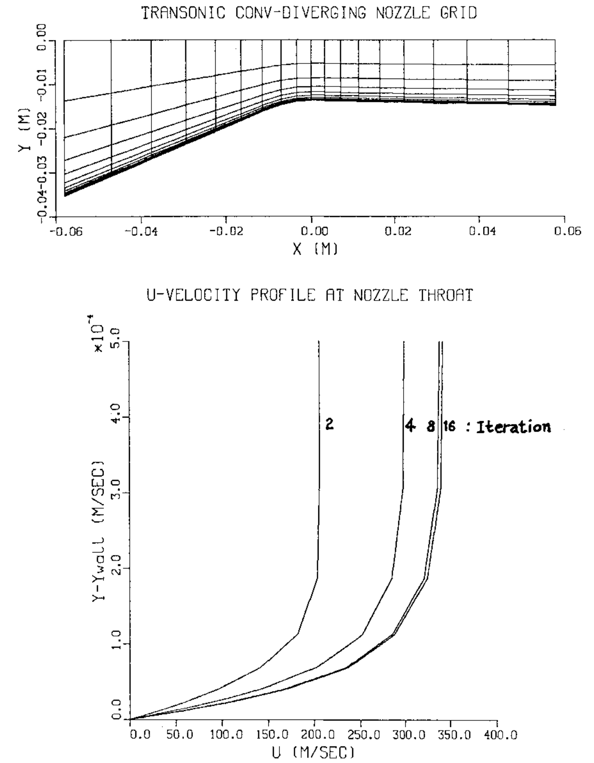

Fig. 1a Convergence of velocity profiles at the nozzle throat: 20×30 mesh.

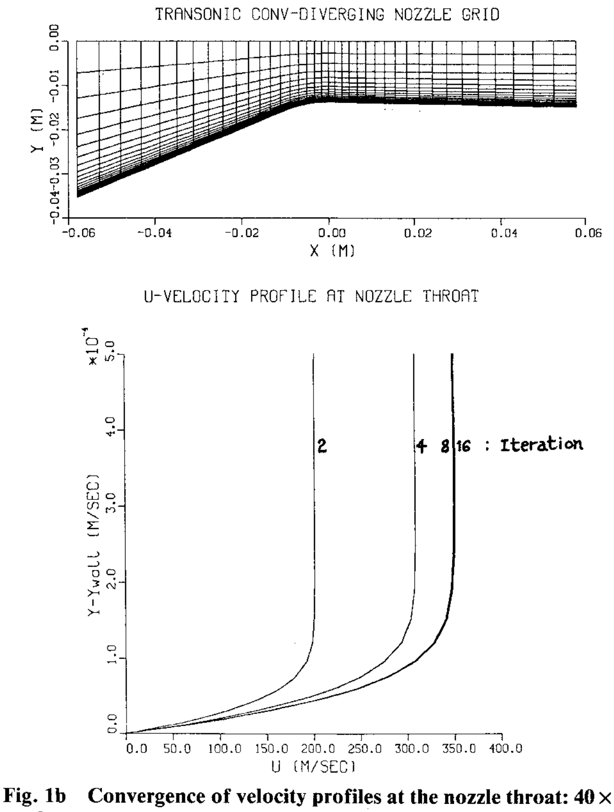

Fig. 1b Convergence of velocity profiles at the nozzle throat: 40×40 mesh.

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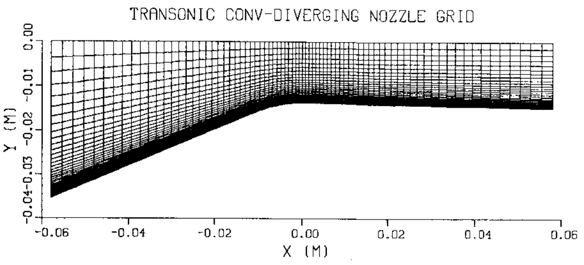

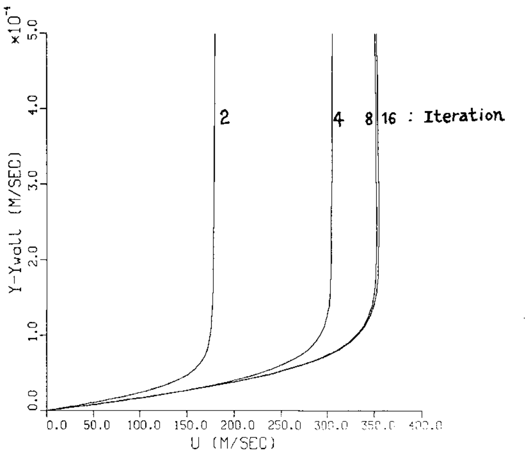

Fig. 1c Convergence of velocity profiles at the nozzle throat: 80×80 mesh.

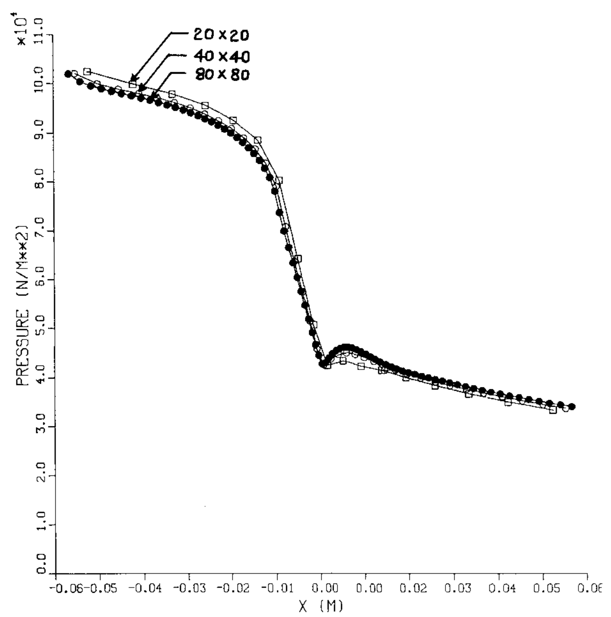

Fig. 2 Pressure along the nozzle wall on 20×20, 40×40, and 80×80 mesh.

in 16 iterations. The numerical results for convergence are shown in Figs. 2 and 3 for the pressure along the nozzle wall and the u component of velocity at the nozzle throat, respectively. They indicate that convergence of the numerical procedure is independent of grid size.

Conclusion

The line Gauss-Seidel relaxation procedures for solving the compressible Navier-Stokes equations show very promising

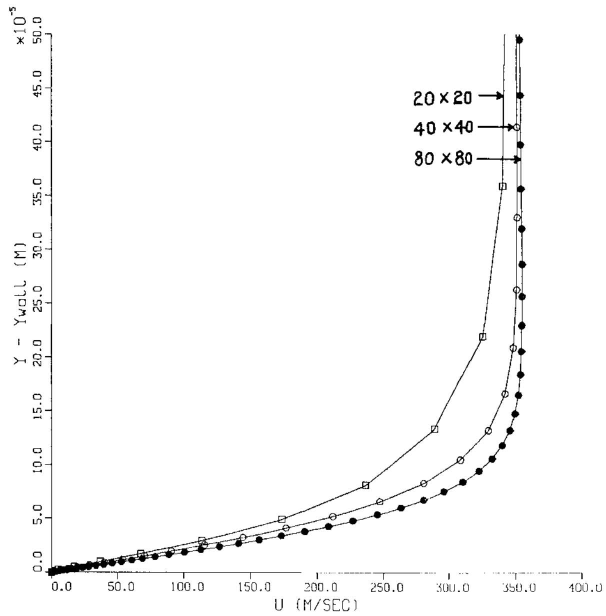

Fig. 3 u component velocity profiles at the nozzle throat on 20×20, 40×40, and 80×80 mesh.

numerical efficiency. The effect of grid size on convergence of the method is tested. The results show that the method is independent of grid size.

Acknowledgment

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References

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Spacing of Streamwise Vortices on Concave Walls

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Introduction

THE stability of streamwise vortices generated on a concave wall by the Görtler mechanism is governed by the parameter

$$G\ddot{o}_\Theta \equiv (U_\infty \Theta / \nu)\ [\Theta / R]^{\frac{1}{2}}$$

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